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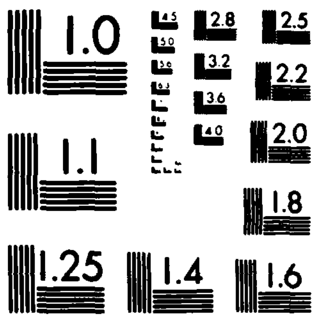

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ADAPTIVE DETECTION OF A KNOWN SIGNAL IN NON-GAUSSIAN NOISE

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ABSTRACT

The design of a locally optimal detector for a known signal in non-Gaussian noise is discussed. The optimal detector nonlinearity is approximated adaptively in the noise pdf tail region, and a polynomial is used to approximate the nonlinearity near the mean. Examples for several different noise environments are presented, showing in these cases that the adaptive detector is able to achieve a high level of performance

1. INTRODUCTION

A binary hypothesis test may be used to model the problem of detecting a known signal in the presence of noise. Let $\vartheta\mathbf{s} = \{s_1, \ldots, s_M\}$ be the known signal sequence with amplitude parameter $\vartheta > 0$, and let $\mathbf{n} = \{n_1, \ldots, n_M\}$ be an iid noise sequence independent of the signal. The detector observes $\mathbf{x}$, a data sequence $\{x_1, \ldots, x_M\}$, and decides between:

$$H_0:\ \mathbf{x} = \mathbf{n}$$
$$H_1:\ \mathbf{x} = \mathbf{n} + \vartheta\mathbf{s}$$

In the framework of Neyman-Pearson hypothesis testing, $\mathbf{x}$ and the multivariate noise density f are used to calculate a likelihood ratio Λ_{NP}. This test statistic and a fixed threshold T_{NP} are compared to arrive at a decision: H_1 is chosen when $\Lambda_{NP} > T_{NP}$, and H_0 is chosen when $\Lambda_{NP} \leq T_{NP}$. More precisely,

$$\Lambda_{NP} = \frac{f_{\mathbf{X}}(\mathbf{x}|H_1)}{f_{\mathbf{X}}(\mathbf{x}|H_0)} = \frac{f_{\mathbf{N}}(\mathbf{x}-\vartheta\mathbf{s})}{f_{\mathbf{N}}(\mathbf{x})} \underset{H_0}{\overset{H_1}{\gtrless}} T_{NP} \qquad (1)$$

Since the noise is iid, and the logarithm function is monotonic, an equivalent test is

$$\lambda_{NP} = \ln \Lambda_{NP} = \sum_{i=1}^{M} g_{NP;i}(x_i) = \sum_{i=1}^{M} \ln \frac{f_n(x_i - \vartheta s_i)}{f_n(x_i)} \underset{H_0}{\overset{H_1}{\gtrless}} t_{NP} = \ln T_{NP} \qquad (2)$$

where f_n is the univariate density of n_i. When the signal is constant, $s_i = s$ for $i = 1, \ldots, M$, and the sequence $\{g_{NP;i}\}$ may be replaced with a zero memory nonlinearity, g_{NP}.

In cases where the signal-to-noise ratio is very small, the test statistic may be calculated via the locally optimal detector. The test becomes

$$\lambda_{LO} = \sum_{i=1}^{M} g_{LO}(x_i) = -\sum_{i=1}^{M} \frac{d}{dx} \ln f_n(x_i) \underset{H_0}{\overset{H_1}{\gtrless}} t_{LO} \qquad (3)$$

Implicit in both detection methods is a requirement that the noise pdf must be known exactly. In general, the noise statistics are not known with precision and the design of the LO or NP detector is not straightforward. Alternative detection strategies are available, and among these are (1) detectors which are robust with respect to deviations from a nominal noise environment [2,8]; (2) nonparametric

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detectors which use only very general information about the underlying noise distribution [3]; and (3) fixed or adaptive suboptimal detectors which have nearly optimal performance [4,7,13]. An additional consideration is that often the noise environment is nonstationary and an adaptive structure is necessary.

2. DETECTION IN NON-GAUSSIAN NOISE

The classical assumption in the design and analysis of detection systems is that the noise is Gaussian. This assumption has attractive features in that the Gaussian model is mathematically tractable, the optimal detector structure is linear, and strong justification for the model is available in the form of the Central Limit Theorem. Measurements of typical noise environments have led to the conclusion that the true noise distribution often is described better by a heavier tailed pdf [5,6,12,13]. This type of noise can be ascribed to a nominal Gaussian environment with a heavy tailed impulsive noise contaminant, and to the fact that a real noise comprises a finite sum of random events; convergence to the Gaussian pdf is not complete for a finite sum, but the sum pdf is most nearly Gaussian near the mean, with the tails converging to the Gaussian pdf only in the limiting case. For example, previous work in detection has taken note of non-Gaussian noise environments, where the noise density is approximately Gaussian near the mean, but heavier tailed away from the mean [1,6,10].

The NP and LO detector nonlinearities related to near-Gaussian heavy tailed densities are typically composed of a linear region surrounded by tails which compress, limit, or even blank large data observations. Work in robust estimation of the mean has similarly suggested that, in heavy tailed noise, a robust estimator should reduce the influence of very large data observations while leaving observations near the mean relatively unchanged [9].

3. A SUBOPTIMAL DETECTOR APPROXIMATION

This paper presents an approach to the design of a noise-adaptive suboptimal detector with these ideas in mind. Attention is focused on the case of LO detection of a constant binary signal in discrete time, with iid noise assumed. Further, the noise pdf is restricted to be unimodal, symmetric about its mean placed at the origin, and to have nonzero support over the entire real line. We are interested in approximating ZNL's loosely specified by the following characteristics:

a) continuous, with continuous low-order derivatives
b) approximately linear at the origin
c) odd symmetric about the origin
d) strictly positive to the right of the origin
e) monotone in the tail regions
f) possess only one local extremum on either side of the origin

Pdf's which are non-Gaussian in the sense previously described can generate LO nonlinearities with these characteristics, which are typical of robust and suboptimal detector nonlinearities, and also of many of the influence curves given for robust estimation procedures [11].

The tail behavior of these LO nonlinearities range from linear for a noise pdf with Gaussian tails, to a limiter for exponentially decreasing pdf tails, to a blanker for algebraically decreasing pdf tails. In general, the heavier tailed the noise density is relative to the Gaussian pdf, the more severely curtailed is the effect of large data observations.

The objective of a noise adaptive nonlinearity, then, should be to relate the ZNL tail behavior to the actually observed noise pdf tail behavior. We propose the following method: It has been reported [12] that the generalized Gaussian density

$$f_c(x) = \frac{\eta c}{2\Gamma(1/c)}\exp(-|\eta x|^c) \tag{4}$$

in certain instances can describe the pdf tails of some physical noise sources. For a noise variance of σ^2, the parameter η is defined by

$$\eta = \left[\frac{\Gamma(3/c)}{\sigma^2\Gamma(1/c)}\right]^{1/2}$$

The corresponding LO nonlinearities, shown in Figure 1, can be written as

$$g_{LO}(x) = c\eta^c\,|x|^{c-1}\,\mathrm{sgn}(x) \tag{5}$$

with the c conveniently parameterizing ZNL tail behavior. Therefore, we model the observed noise pdf tails via the generalized Gaussian family. The suboptimal LO nonlinearity thus will have power law tails described by

$$\hat{g}_{LO}(x) = \hat{c}\eta^{\hat{c}}\,|x|^{\hat{c}-1}\,\mathrm{sgn}(x) \quad \text{for}\,|x| > x_0 \tag{6}$$

It is necessary to find a value $\hat{c}$ such that $f_{\hat{c}}$ is a good approximant to the tail behavior of f_n, the true but unknown underlying noise density. A simple way to do this is to equate the tail probability of $f_{\hat{c}}$ with the observed tail mass

$$\hat{P}_T = \frac{1}{N}\sum_{i=1}^{N} I_{(T,\infty)}(|r_i|) \tag{7}$$

Here, I is the indicator function and r_i are the noise observations presumed available from a noise reference channel. The exponent c can be estimated then by finding $\hat{c}$ such that

$$2\int_T^{\infty} f_{\hat{c}}(x)dx = \hat{P}_T \tag{8}$$

For the purpose of calculating η, the observed noise variance $\sigma_N{}^2$ is used. The estimate $\hat{c}$ is defined by the implicit integral in (8), so it is desirable to derive a simpler explicit relationship

$$\hat{c} = h_T(\hat{P}_T) \tag{9}$$

One obvious method is to first calculate P_T as a function of c, and then interpolate this tabulation to approximate (9). This tabulated version of h_T is shown in Figure 2.

With σ^2 fixed and c small, the value of η, a scale factor, becomes large. Therefore, even though $f_c(x)$ approaches zero asymptotically at a much slower rate than the Gaussian pdf as $|x|$ gets large, the total probability mass in the tails is quite small. As a result, h_T is multiple valued, the density is extremely peaked, and the LO nonlinearity has a discontinuity at the origin. Clearly, for $c < 1$, the requirement of near-linearity at the origin is not met by (5). Since the objective in using the generalized Gaussian pdf is to relate the tail heaviness of an observed noise to a parameter governing the shape of the ZNL tail, we replace the anomalous behavior of the true function h with a simple linear relation

$$\hat{h}_T(\hat{P}_T) = a\hat{P}_T + b \tag{10}$$

where a and b are chosen to approximate (9) for a particular value of T. This approximation is plotted as the broken line on Figure 2. The values for a and b are chosen so that when $\hat{P}_T$ corresponds to Gaussian or exponential noise tails, (10) gives $\hat{c} = 2$ and $\hat{c} = 1$, respectively. Note that the linear relation allows the value of $\hat{c}$ to be negative for large tail probabilities.

The tail measurement threshold T must be chosen prior to estimating parameters a and b. One way to pick T is to choose a value which minimizes

$$E_c\left\{\operatorname{var}\hat{c}\right\} = a^2 E_c\left\{\frac{P_T(1-P_T)}{N}\right\} \tag{11}$$

where N is the number of noise observations. For c uniformly distributed on the interval $[1,2)$, and P_T the tail probability of $f_c(x)$ for $|x| > T$, the constant $T = 3\sigma$ approximately minimizes (11).

The LO nonlinearities of the generalized Gaussian family have desirable tail behavior, but for small values of $\hat{c}$ the behavior does not meet the constraint of linearity near the origin. To eliminate this behavior, the ZNL needs modification in the region near the origin. A way in which to do this is to replace $\hat{g}_{LO}(x)$, for x near zero, with a function that fits the given desired characteristics (a-f). A suitable family of functions are polynomials $p(x)$ with the following characteristics:

$$p(x) = \alpha_3 x^3 + \alpha_2 x^2 + \alpha_1 x + \alpha_0 \quad \text{for} \quad 0 \le x \le x_0$$
$$p(0) = 0$$
$$p(|\pm x_0|)\operatorname{sgn}(\pm x_0) = \hat{g}_{LO}(\pm x_0)$$
$$p'(|\pm x_0|) = \hat{g}'_{LO}(\pm x_0)$$
$$p''(|\pm x_0|)\operatorname{sgn}(\pm x_0) = \hat{g}''_{LO}(\pm x_0)$$

The choice of tail behavior via $\hat{c}$ and the point x_0 completely specify $p(x)$. The method for choosing $\hat{c}$ has already been specified, so the choice of x_0 is the remaining free parameter.

A method equivalent to choosing the proper x_0 is to choose an arbitrary x_0 and scale the input to the ZNL with a factor ν. It is reasonable to choose ν to maximize the efficacy of the ZNL. For an arbitrary nonlinearity q, efficacy as a function of ν can be defined as

$$\xi_q(\nu) = \frac{\nu^2 E_f^2[q'(\nu x)]}{E_f[q^2(\nu x)] - E_f^2[q(\nu x)]} \tag{12}$$

where E_f refers to the expectation with respect to the density function $f(x)$, and q' refers to the first derivative of q.

At this point, specification of the suboptimal nonlinearity $\hat{g}$ is complete, and can be written as

$$\hat{g}(\nu x) = \begin{cases} p(|\nu x|)\operatorname{sgn}(\nu x) & \text{if } |\nu x| \le x_0 \\ \hat{c}\,|\nu x|^{\hat{c}-1}\operatorname{sgn}(\nu x) & \text{if } |\nu x| > x_0 \end{cases} \tag{13}$$

Figures 3a-3c gives some examples of the types of nonlinearities available using this approximation method.

4. EXAMPLES

Analytic

We will now present examples of the use of $\hat{g}$ in approximating some known optimal LO nonlinearities.

The first comparison is between the approximate and exact versions of LO nonlinearities for the generalized Gaussian family. The exponent $\hat{c}$ is given by (10), after using the exact value c in (6) to obtain P_T. Since this is an analytical example, and the true noise density is known, numerical methods can be used to obtain ν^*, the value of ν which maximizes $\xi_g(\nu)$. The performance of the suboptimal ZNL relative to the LO nonlinearity can be measured by asymptotic relative efficiency, which is given easily in terms of efficacy as

$$\mathrm{ARE}_{\hat{g},g_{LO}} = \frac{\xi_{\hat{g}}}{\xi_{g_{LO}}} \tag{14}$$

Figure 4 compares the performance of $\hat{g}$, the LO detector and a linear detector (ld), in terms of $\mathrm{ARE}_{\hat{g},g_{LO}}$ and $\mathrm{ARE}_{g_{LO},ld}$. The suboptimal nonlinearity performs quite well for the range $1 \le c \le 2$, but for $c < 1$, performance deteriorates. This is easily explainable, since for small c, the LO nonlinearity output approaches $\pm\infty$ for inputs near zero, while the approximation method requires $\hat{g}$ to pass through the origin

Another family of heavy tailed pdf's is the Johnson S_u family. If X is distributed as an $N(0,\sigma^2)$ random variable, and we define a new random variable

$$y = \lambda \sinh\left[\frac{x}{\delta}\right] \tag{15}$$

then the density of Y has unity variance, and belongs to the Johnson S_u family, given by

$$f_\delta(y) = \frac{\delta}{\lambda\sqrt{2\pi}}\left[\left[\frac{y}{\lambda}\right]^2 + 1\right]^{-\frac{1}{2}} \exp\left[-\frac{\delta}{2}\sinh^{-1}\left[\frac{y}{\lambda}\right]^2\right] \tag{16}$$

with

$$\lambda = \left[\frac{2\sigma^2}{\exp(2/\delta^2)-1}\right]^{\frac{1}{2}} \tag{17}$$

The parameter δ controls the tail heaviness. As $\delta \to \infty$, the pdf tails become progressively lighter, and approach Gaussian tails in the limit.

Since f_δ is given and known, $\hat{P}_T$ can be calculated from (8), and (10) gives $\hat{c}$. Again, numerical methods can find the ν^* which maximizes (12). Some representative LO nonlinearities and suboptimal approximations are given for various values of δ in Figure 5, and Figure 6 presents the performance comparison of $\hat{g}$, g_{LO}, and ld. For this set of densities, the approximation method works quite well. Over the range $.8 \le \delta \le \infty$, the minimum of $\mathrm{ARE}_{\hat{g},g_{LO}}$ is .989, (occurring for $\delta = .8$). This means that only a small performance penalty would be incurred if $\hat{g}$ were to replace the LO detector. As a final comment, it should be observed that unlike the generalized Gaussian family, the Johnson S_u family fulfills the characteristics of a near Gaussian pdf given earlier.

A third family of heavy tailed densities was also investigated. This third family is the ε-contaminated Gaussian mixture, and in this case, the Gaussian-Gaussian mixture. The pdf of this noise can be written as

$$f_\varepsilon(x) = (1-\varepsilon)f_0(x) + \varepsilon f_1(x) \tag{18}$$

where f_0 represents the pdf of an $N(0,1)$ distributed random variable, and f_1 represents the pdf of an $N(0,\sigma^2)$ random variable, with σ^2 large. The parameter ε controls the degree to which f_1 contaminates the nominal density f_0, and is typically taken to be small. The performance of the suboptimal ZNL was calculated for some specific cases, and the results appear in Table 1. Figure 7 shows the comparison in the form of two LO nonlinearities and the corresponding approximations. The performance of the approximations in this case is not as good as the approximations under the Johnson S_u family. Once again not all the assumptions are met by the ε-contaminated Gaussian-Gaussian mixture on the form of the LO nonlinearity, as there is more than one local extremum on each side of the origin.

Table 1
Performance of g_{LO} and $\hat{g}$ in ε-contaminated Gaussian-Gaussian noise

σ^2	ε	x_0	$\hat{c}$	ν^*	$ARE_{\hat{g},g_{LO}}$	$ARE_{g_{LO},ld}$
20	.05	3	.734	.767	.952	1.72
20	.20	3	-.196	.821	.960	3.10
5	.05	3	1.54	.957	.977	1.08
5	.20	3	.996	.957	.983	1.24

The results so far show that, in several analytical examples, it is possible to achieve reasonably good performance from this simple approximation.

Simulation

To see how well this system might work in practice, some actual physical noise was used to drive the system. The noise was collected underneath the Arctic ice pack, and details may be found in [13]. The noise data is highly nonstationary; a background Gaussian noise is abruptly interrupted with segments of a very heavy tailed noise generated by cracking of the Arctic ice pack. The kurtosis for each consecutive block of 1024 noise samples was calculated, and blocks with a kurtosis exceeding 4 were selected. To get a more nearly stationary noise for driving the system, the data in each block was normalized to unit variance, randomly permuted, and the blocks were then concatenated, thus simulating the output of a physical noise source. This noise was used as the input for the adaptive system. The exponent $\hat{c}$ was estimated from the running estimate of $\hat{P}_T$, which converges to the true tail probability as $N \to \infty$. The scale parameter ν cannot be found via numerical methods, since in this case the simulated system has no knowledge of the true density generating the noise observations. However, the Kiefer-Wolfowitz stochastic approximation method can be used to find the value of ν^* which maximizes $\xi_{\hat{g}}$. The convergence rate towards ν^* is fixed by the particulars of the SA algorithm, and no formal attempts were made to optimize its performance.

Figure 8 shows the running estimate of $\hat{c}$ and ν as a function of sample number. Figure 9 shows the estimated efficacy of $\hat{g}$ for each 1024 sample block, and the cumulative estimate of efficacy starting from the first noise observation. At the end of the simulation, it was assumed that the approximated ZNL was as near optimal as possible. The final ZNL is shown in Figure 10. The efficacy of this suboptimal detector was calculated using the empirical distribution of the entire noise observation set, and it was found to be 1.39. The true distribution of the noise is not known, and therefore it is not possible to calculate the efficacy of the true LO nonlinearity. However, it is possible to conclude that the adaptive system was able to use the noise observations and adapt the detector structure in a constructive way. The ARE of the approximate ZNL relative to the linear detector is given simply by the efficacy in this case, and therefore the adaptive ZNL shows an improved performance over the linear detector.

5. CONCLUSION

The conclusion to be drawn from this study is that it is possible to implement an adaptive detector nonlinearity using fairly simple techniques. The estimate of tail behavior is quite simple: merely a measurement of the relative number of samples exceeding a specified threshold. This apparently gives enough information about tail behavior of the true noise density, so that even a crude approximation to the true ZNL in the tail regions results in fairly good performance. A more sophisticated estimate of $\hat{c}$ might improve the performance $\hat{g}$. It would be interesting to discover how much additional complexity any resulting performance gain could justify.

Some natural simplifications to this approximation method suggest themselves for investigation. One is to replace the central region polynomial in the ZNL simply by a linear connection between the tail approximants at $\pm x_0$. A further simplification would be to replace the power-law tails with piecewise linear approximations, possibly even a single straight line. Some work done by others [7] in approximation of LO nonlinearities suggest that even very simple approximants of the optimal nonlinearity have the potential to achieve performance which is acceptably near the optimal.

ACKNOWLEDGEMENT

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REFERENCES

[1] J. H. Miller and J. B. Thomas, "Detectors for Discrete-Time Signals in Non-Gaussian Noise," *IEEE Trans. Inform. Theory*, vol. IT-18, no. 2, pp. 241-250, March 1972.

[2] P. J. Huber, "A Robust Version of the Probability Ratio Test," *Ann. Math. Statist.*, vol. 36, pp.1753-1758, Dec. 1965.

[3] J. B. Thomas, "Nonparametric Detection," *Proc. IEEE*, vol.58, no. 5, pp.623-631, May 1970.

[4] J. W. Modestino, "Adaptive Detection of Signals in Impulsive Noise Environments," *IEEE Trans. Commun.*, vol. COM-25, no. 9, pp. 1022-1026, Sept. 1977.

[5] S. L. Bernstein, *et al*, "Long-Range Communications at Extremely Low Frequencies," *Proc. IEEE*, vol. 62, no. 3, pp. 292-312, March 1974.

[6] A. D. Spaulding and D. Middleton, "Optimum Reception in an Impulsive Interference Environment - Part I: Coherent Detection," *IEEE Trans. Commun.*, vol COM-25, no. 9, pp. 924-934, Sept. 1977.

[7] J. H. Miller and J. B. Thomas, "Robust Detectors for Signals in Non-Gaussian Noise," *IEEE Trans. Commun.*, vol. COM-25, no. 7, pp. 686-690, July 1977.

[8] A. H. El-Sawy and V. D. Vandelinde, "Robust Detection of Known Signals," *IEEE Trans. Inform. Theory*, vol. IT-23, no. 6, pp. 722-727, Nov. 1977.

[9] F. R. Hampel, "The Influence Curve and its Role in Robust Estimation," *Jour. Amer. Stat. Assoc.*, vol. 69, no. 346, pp. 383-393, June 1974.

[10] R. F. Ingram and R. Houle, "Performance of the Optimum and Several Suboptimum Receivers for Threshold Detection of Known Signals in Additive, White, Non-Gaussian Noise," Technical Report 6339, Naval Underwater Systems Center, New London, CT, Nov. 24, 1980.

[11] D. F. Andrews, *et al*, *Robust Estimates of Location*, Princeton University Press, Princeton, NJ, 1972.

[12] A. D. Watt and E. L. Maxwell, "Measured Statistical Characteristics of VLF Atmospheric Noise," *Proc. Inst. Radio Engineers*, vol. 45, no. 1, pp 55-62, Jan. 1957.

[13] R. F. Dwyer, "Arctic Ambient Noise Statistical Measurement Results and Their Implications to Sonar Performance Improvements," Technical Report 6739, Naval Underwater Systems Center, New London, CT, May 5, 1982.

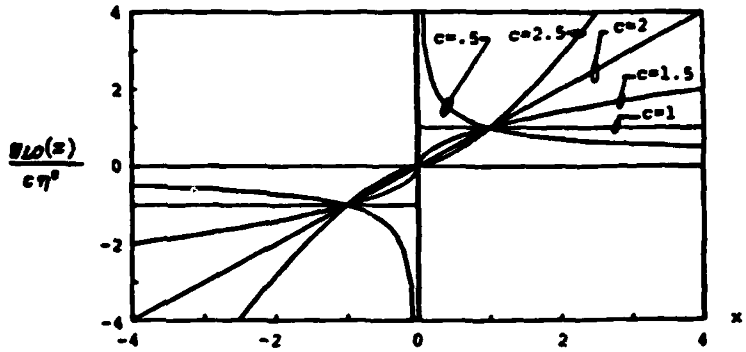

Figure 1. Locally optimal nonlinearities for the generalized Gaussian family.

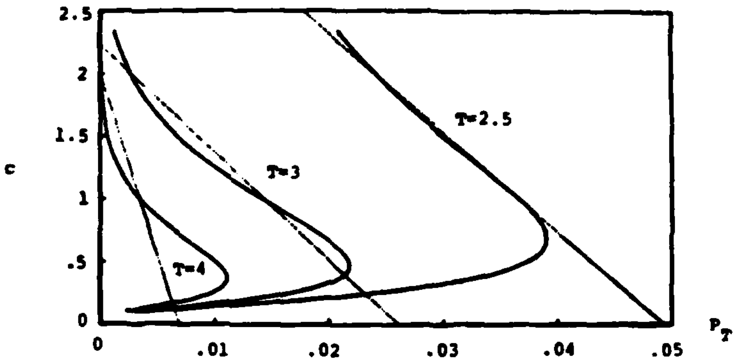

Figure 2. The exact relation $c = h_T(P_T)$ (solid line), and its approximation $\hat{c} = \hat{h}_T(P_T)$ (broken line) for various T.

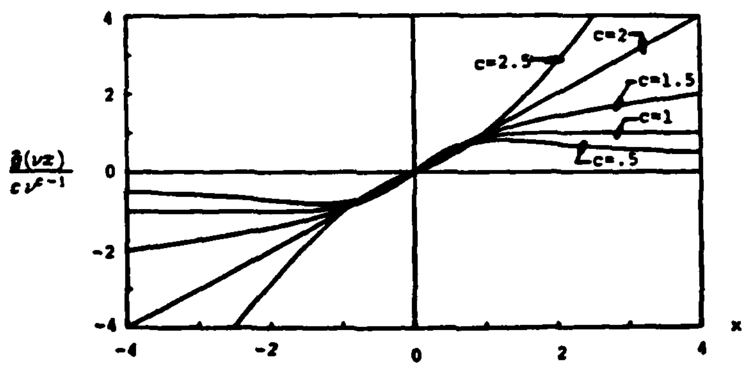

Figure 3a. The ZNL $\hat{g}$ for $z_0 = 3$, $\nu = 1$, various c.

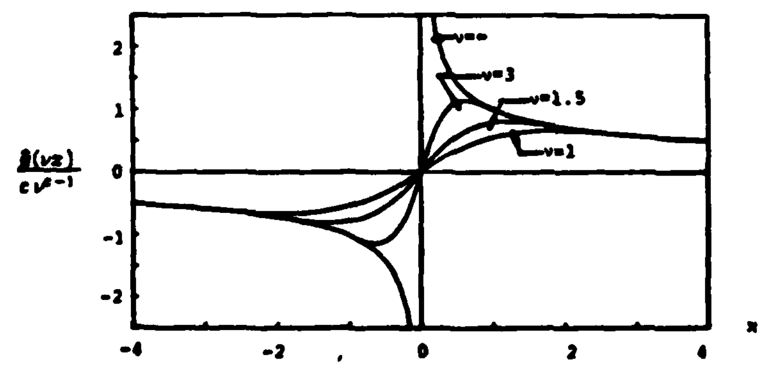

Figure 3b. The ZNL $\hat{g}$ for $z_0 = 3$, various ν, $c = .5$.

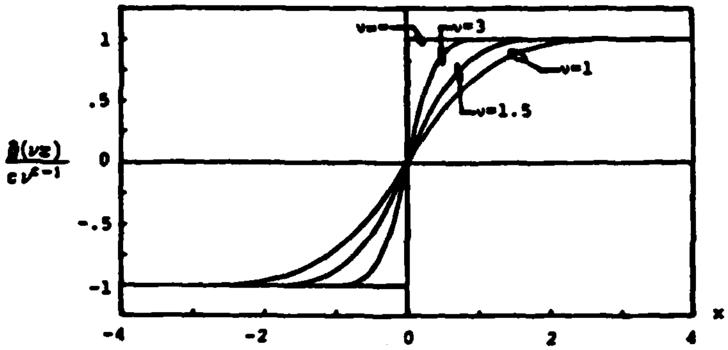

Figure 3c The ZNL $\hat{g}$ for $x_0 = 3$, various ν, $c = 1$.

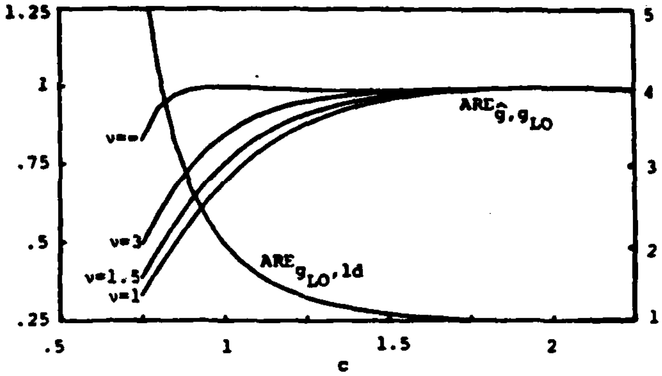

Figure 4. Performance of $\hat{g}$ and g_{LO} versus c for the generalized Gaussian family. $\mathrm{ARE}_{\hat{g},g_{LO}}$ left scale; $\mathrm{ARE}_{g_{LO},ld}$ right scale

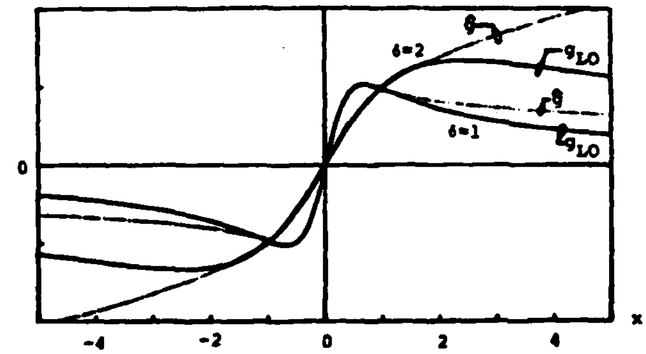

Figure 5. g_{LO} (solid line) and $\hat{g}$ (broken line) for Johnson S_u family, on different vertical scales for comparison. For $\delta = 1$, $\hat{c} = .752$, $\nu^* = 3.26$, $x_0 = 3$. For $\delta = 2$, $\hat{c} = 1.46$, $\nu^* = 1.88$, $x_0 = 3$.

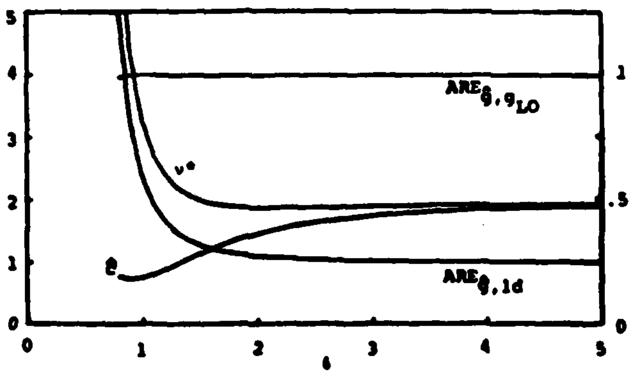

Figure 6. Performance of $\hat{g}$ and g_{LO} versus δ for the Johnson S_u family. $\mathrm{ARE}_{\hat{g},g_{LO}}$, right scale; all others on left scale

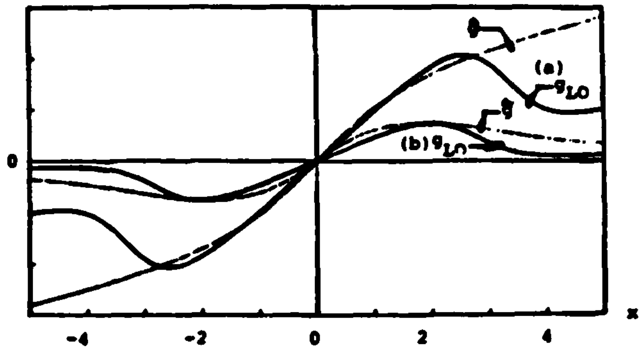

Figure 7. g_{LO} (solid line) and $\hat{g}$ (broken line) for Gaussian-Gaussian mixture, on different vertical scales for comparison. (a) $\epsilon = .05$ $\sigma_1^2 = 5$; (b) $\epsilon = .20$ $\sigma_1^2 = 20$

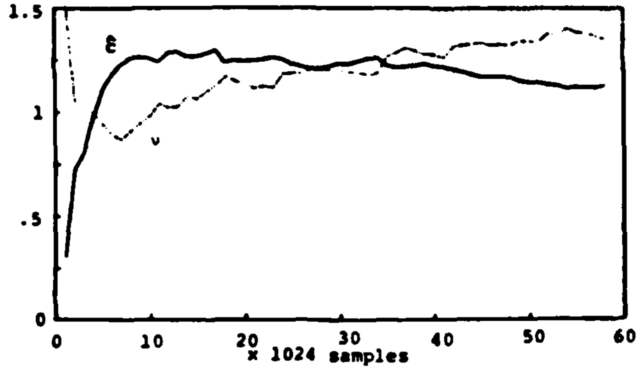

Figure 8. ZNL parameters versus observation number for heavy tailed Arctic ice noise.

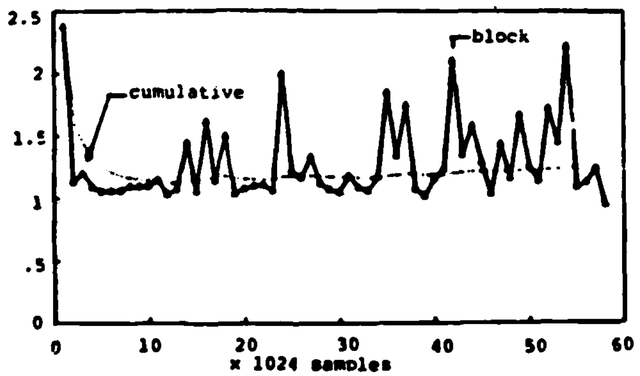

Figure 9. Estimated performance of $\hat{g}$ for the heavy tailed Arctic ice noise

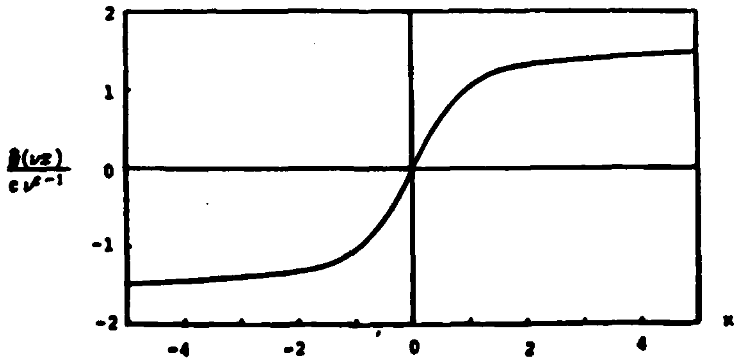

Figure 10. Estimated ZNL at end of Arctic ice noise simulation $\hat{\epsilon} = 1.13$, $\nu^* = 1.35$, $x_0 = 3$